# Interplay between QCD and nuclear responses.

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**Abstract**

We establish the interrelation between the QCD scalar response of the nuclear medium and its response to a scalar probe coupled to nucleons, such as the scalar meson responsible for the nuclear binding. The relation that we derive applies at the nucleonic as well as at the nuclear levels. Non trivial consequences follow. One concerns the scalar QCD susceptibility of the nucleon. The other opens the possibility of relating medium effects in the scalar meson exchange of nuclear physics to QCD lattice studies of the nucleon mass.

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## 1 Introduction

The spectrum of scalar-isoscalar excitations is quite different in the vacuum and in the nuclear medium. In the second case it includes low lying nuclear excitations and also two quasi-pion states *i.e.* pions dressed by particle-hole excitations. All these lie at lower energies than the vacuum scalar excitations which start at $2\,m_\pi$. We have shown in previous works [1, 2, 3, 4] that this produces a large increase of the magnitude of the scalar QCD susceptibility over its vacuum value. We have expressed the origin of this increase as arising from the mixing of the nuclear response to a scalar probe coupled to nucleonic scalar density fluctuations into the QCD scalar response.

It is natural to investigate also the reciprocal problem of the influence of the QCD scalar response to a probe which couples to the quark density fluctuations on the ordinary nuclear scalar response of nuclear physics, which is the object of the present work. We will study this influence not only for what concerns the nuclear excitations but also for a single nucleon for which only nucleonic excitations are involved. If this influence indeed exists, does it lead to non-trivial observable consequences ? We will show that this is the case, with two main applications. One concerns the QCD scalar susceptibility of a single nucleon. The second is the possibility to infer medium effects in the propagation of the scalar meson which binds the nucleus from QCD results, such as the lattice ones on the evolution of the nucleon mass with the pion mass.

Our article is organized as follows. In section **2** we remind the mechanisms responsible for the mixing of the nuclear response into the QCD scalar susceptibility. We illustrate it in the framework of a nuclear chiral model with a scalar and vector meson exchange. We show that this mutual influence also exists at the nucleonic level. In section **3** we discuss the influence of the quark structure of the nucleon on the scalar response of nuclear physics in a general framework which is able to incorporate also confinement effects.

# 2 Mutual influence of the scalar QCD response and nuclear physics response

## 2.1 Study in a nuclear chiral model

We first remind how the usual nuclear physics response to a scalar field enters in the QCD susceptibility. For this, following ref. [2], we start from the expression of the modification of the quark condensate in the nuclear medium, $\Delta\langle\bar{q}q\rangle(\rho) = \langle\bar{q}q\rangle(\rho) - \langle\bar{q}q\rangle_{vac}$. We first use, as in [2], its expression for a collection of independent nucleons :

$$\Delta\langle\bar{q}q\rangle(\rho) = Q_S\,\rho_S. \tag{1}$$

where $\rho_S$ is the scalar density of nucleons related to the chemical potential $\mu$ by :

$$\rho_S = 4\int\frac{d^3p}{(2\pi)^3}\,\frac{M}{E_p}\,\Theta(\mu - E_p). \tag{2}$$

We have introduced the scalar charge of the nucleon, $Q_S$, proportional to the volume integral of the nucleon scalar density of quarks. It is related to the nucleon sigma commutator $\sigma_N$ and the current quark mass, $m_q$, by :

$$Q_S = \frac{\sigma_N}{2m_q} = \int d^3r\, N|\bar{q}q(\vec{r}) - \langle\bar{q}q\rangle_{vac}|N\rangle. \tag{3}$$

The susceptibility of the nuclear medium, $\chi_S^A$, is the derivative of the quark scalar density with respect to the quark mass. We define it in such a way that it represents a purely nuclear contribution with the vacuum susceptibility substracted off :

$$\chi_S^A = \left(\frac{\partial\Delta\langle\bar{q}q(\rho)}{\partial m_q}\right)_\mu = \left(\frac{\partial(Q_S\,\rho_S)}{\partial m_q}\right)_\mu. \tag{4}$$

It contains two terms. One arises from the derivative of $Q_S$, which by definition is the free nucleon QCD scalar susceptibility, $\chi_S^N = \partial Q_S/\partial m_q$. The second one involves the derivative of the nucleon density. This term itself is built of two pieces, one involves antinucleon excitations and is small [2]. The other one, which is larger, involves the nuclear response $\Pi_0 = -2M_N p_F/\pi^2$. In this case it is the free Fermi gas one since no interactions between nucleons have been introduced. The result of this derivation is contained in the following equation:

$$\chi_S^A = \rho_S\,\chi_N^S\,+\,2\,Q_S^2\,\Pi_0\,. \tag{5}$$

The nuclear susceptibility is thus the sum of a one-body term and of a term due the nuclear excited states, the p-h excitations. This decomposition survives the introduction of the interaction, as will be shown next. In this case the free p-h polarization propagator is replaced by the full RPA one, while the free nucleon susceptibility can undergo medium modifications and become dependent on the density.

The previous result has been generalized in ref. [3] to an assembly of nucleons interacting

through a scalar and a vector meson exchanges, working at the mean field level as in relativistic mean field theories. In this work the condensate was obtained as the derivative of the grand potential with respect to the quark mass (Feynman-Hellman theorem) and the susceptibility as the derivative of the condensate, both being taken at constant chemical potential. The result is [3] :

$$\chi_S = \left(\frac{\partial \langle \bar{q}q \rangle}{\partial m_q}\right)_\mu \simeq -2\,\frac{\langle \bar{q}q \rangle_{vac}^2}{f_\pi^2}\,\left(\frac{\partial \bar{S}}{\partial c}\right)_\mu . \tag{6}$$

$\bar{S} \equiv f_\pi + \bar{s}$ is the expectation value of the chiral invariant scalar field and $c = f_\pi\, m_\pi^2$ is the symmetry breaking parameter of the model used in [3]. The quantity $\left(\partial \bar{S}/\partial c\right)_\mu$ is related to the in-medium sigma propagator :

$$\left(\frac{\partial \bar{S}}{\partial c}\right)_\mu = -D_\sigma^* = \frac{1}{m_\sigma^{*2}} - \frac{g_S^2}{m_\sigma^{*2}}\,\Pi_S(0)\,\frac{1}{m_\sigma^{*2}} \tag{7}$$

where $\Pi_S(0)$ is the full scalar polarization propagator, related to the bare one, $\Pi_0$ by :

$$\Pi_S(0) = \frac{M_N^*}{E_F^*}\,\Pi_0(0)\,\left[1 - \left(\frac{g_\omega^2}{m_\omega^2}\,\frac{E_F^*}{M_N^*} - \frac{g_S^{*2}}{m_\sigma^{*2}}\,\frac{M_N^*}{E_F^*}\right)\Pi_0(0)\right]^{-1} . \tag{8}$$

In the equations above, $m_\sigma^*$ is the in-medium sigma mass, which is obtained from the second derivative of the energy density with respect to the order parameter :

$$m_\sigma^{*2} = \frac{\partial^2 \varepsilon}{\partial \bar{s}^2} = V''(\bar{s}) + \frac{\partial\,(g_S\,\rho_S)}{\partial \bar{s}} = m_\sigma^2\left(1 + \frac{3\bar{s}}{f_\pi} + \frac{3}{2}\left(\frac{\bar{s}}{f_\pi}\right)^2\right) \tag{9}$$

where the potential $V$ responsible for the spontaneous symmetry breaking is the standard quartic one of the linear sigma model. In the very last expression of eq. (9) we have omitted, as in ref.[3], the small antinucleon contribution embedded in the factor $\partial\rho_S/\partial\bar{s}$. Moreover since for the moment we do not consider, contrary to ref.[3], the scalar response of the nucleon due to confinement, we also ignore the medium renormalization of $g_S$. The mean scalar field $\bar{s}$ being negative, the term linear in $\bar{s}$ lowers the sigma mass by an appreciable amount ($\simeq 30$ % at $\rho_0$). This is the chiral dropping associated with chiral restoration [5] and arising from the $3\sigma$ interaction as depicted in fig 1.

Since we are interested only in the medium effects the vacuum value of the quantity $\left(\partial \bar{S}/\partial c\right)_\mu = 1/{m_\sigma}^2$ has to be subtracted off in eq. (7) and the purely nuclear suceptibility, $\chi_S^A$, writes :

$$\chi_S^A \;=\; 2\,\frac{\langle \bar{q}q \rangle_{vac}^2}{f_\pi^2}\left[\frac{3\,\bar{s}/f_\pi + \frac{3}{2}\,(\bar{s}/f_\pi)^2}{m_\sigma^{*2}} + \frac{g_S^2}{m_\sigma^{*2}}\,\Pi_S(0)\,\frac{1}{m_\sigma^{*2}}\right] . \tag{10}$$

We see that $\chi_S^A$ receives two types of contributions, the second being proportionnal to the full RPA scalar response $\Pi_S$. The corresponding proportionality factor $r$ between

this second contribution and $\Pi_S$ writes, to leading order, *i.e.*, neglecting the medium modification of the sigma mass :

$$r = 2\, g_S^2\, \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^2\, m_\sigma^4} \simeq 2\, (Q_S^s)^2 \tag{11}$$

where we have introduced the nucleon scalar charge $Q_S^s$ from the scalar field, defined below. In the sigma model the free nucleon sigma commutator is the sum of two contributions, one arising from the pion cloud, which depends on the mean value of the squared pion field, *i.e.*, on the scalar number of pions in the nucleonic cloud. In the mean field approximation where pion loops are ignored this term does not appear. The other one, $Q_S^s$, arises from the scalar meson [6, 7, 8] and is linear in the $\sigma$ field :

$$Q_S^s = \frac{\sigma_N^s}{2m_q} = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi} \int d^3r\, \langle N|\sigma(\vec{r})|N\rangle = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\, \frac{g_S}{m_\sigma^2} \tag{12}$$

which establishes relation (11) if we ignore the in-medium modification of $Q_S^s$, *i.e.*, the difference beween $m_\sigma^*$ and $m_\sigma$.

We now turn to the first part of $\chi_S^A$ which depends on the average scalar field $\bar{s}$. In the low density limit, $\bar{s}$ reduces to $\bar{s} = -g_S\, \rho_S/m_\sigma^2$, and we can ignore the term in $\bar{s}^2$ as well as the difference beween $m_\sigma^*$ and $m_\sigma$. In this limit the first term in the expression (10) of $\chi_S^A$ is linear in the density. If we classify it in the decomposition of eq. (5) for $\chi_S^A$, it obviously belongs to the individual nucleon contribution, $\rho_S\, \chi_S^N$, to the nuclear susceptibility. Writing the linear term explicitly in eq. (10) we deduce the free nucleon scalar susceptibility from the scalar field, $(\chi_S^N)^s$ :

$$(\chi_S^N)^s \;=\; -2\, \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^3}\, \frac{3\, g_S}{m_\sigma^4}, \tag{13}$$

which is negative (*i.e*, it favors an increase in magnitude of the field, similar to paramagnetism). It has been obtained here from the low density expression of $\chi_S^A$. In fact it can also be obtained directly as the derivative with respect to the quark mass of $Q_S^s$, the part of the nucleon scalar charge originating in the scalar field written in eq. (12) :

$$(\chi_S^N)^s \;=\frac{\partial Q_S^s}{\partial m_q} = \frac{\partial}{\partial m_q}\left(-\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\, \frac{g_S}{m_\sigma^2}\right). \tag{14}$$

Using the fact that, in the model, $\langle\bar{q}q\rangle_{vac}/f_\pi$ does not depend on $m_q$, only the derivative of the sigma mass with respect to $m_q$ enters which, according to the Feynman-Hellmann theorem, is linked to the sigma commutator, $\sigma_\sigma$, of the $\sigma$. In the linear sigma model the derivative with respect to the quark mass is replaced by the derivative with respect to the symmetry breaking parameter, $c = f_\pi\, m_\pi^2$, keeping the other original parameters of the model, $\lambda$ and $v$, constant. The result is :

$$\sigma_\sigma = m_q\, \frac{\partial m_\sigma}{\partial m_q} = \frac{3}{2}\, \frac{m_\pi^2}{m_\sigma}. \tag{15}$$

When inserted in eq. (14), it leads for $(\chi_S^N)^s$ to the expression of eq. (13).

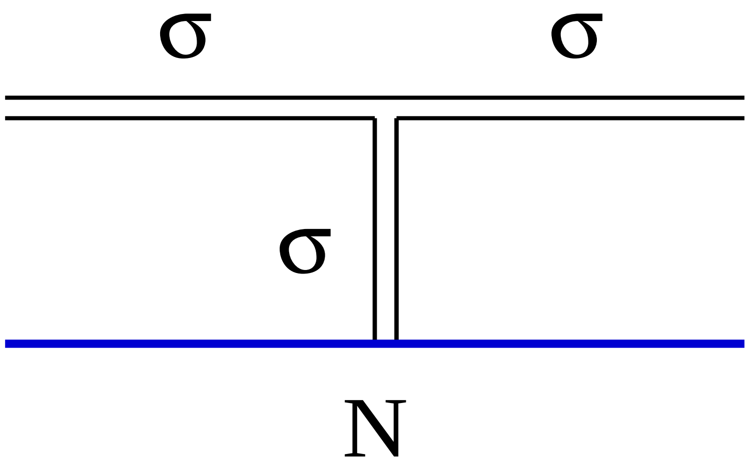


Figure 1: Contribution to the sigma-nucleon scattering amplitude responsible for the lowering of the sigma mass in the medium.

We now turn to the scattering amplitude for the sigma meson on the nuclear system. In the same framework we will first show that the amplitude for the scattering of the scalar meson on the nucleon has the same relation to the nucleonic susceptibility as the case for the nuclear excitation part. Indeed in the expression (9) of $m_\sigma^{*2}$ the term linear in density is obtained from the low density expression : $3\,\bar{s}\,m_\sigma^{*2} \simeq -(3\,g_S/f_\pi)\,\rho_S$. It represents an optical potential for the sigma propagation. The corresponding $\sigma N$ scattering amplitude, $T_{\sigma_N}$, which can also be evaluated directly from the graph of fig. 1, is equal to :

$$T_{\sigma N} = -3\,g_S/f_\pi. \tag{16}$$

We are now in a situation to relate the nucleon scalar susceptibility (eq. (13)) to the sigma-nucleon amplitude of eq. (16), with the result :

$$(\chi_S^N)^s = \frac{2\,(Q_S^s)^2}{g_S^2}\,T_{\sigma N}. \tag{17}$$

The proportionality factor, $2\,(Q_S^s)^2/g_S^2$, is the same as for the purely nuclear excitations. The quantity $g_S$ which appears in this factor factor is due to the $\sigma NN$ coupling constant. Adding the two effects from the nucleonic and nuclear excitations the total QCD scalar susceptibility of the nuclear medium (vacuum value substracted) can therefore be related to the total response, $T^A$, to the scalar field through :

$$\chi_S^A = \frac{2\,(Q_S^s)^2}{g_S^2} T^A \tag{18}$$

where the two members include both the individual nucleon contribution and the one arising from the nuclear excitations,with :

$$T^A = \rho_S\,T_{\sigma N} \;+\; g_S^2\,\Pi_{SS}. \tag{19}$$

The last term on the r.h.s. represents the (in-medium corrected) Born part of the $\sigma N$ amplitude while the first piece represents the non-Born part linked to nucleonic excitations.

Thus there exists a universal scaling factor between the responses of a nuclear or nucleonic system to probes which couple either to the nucleon scalar density fluctuations or to the quark ones. This relation has allowed us to infer the existence of a contribution to the QCD nucleon scalar susceptibility linked to the scalar meson. To the best of our knowledge this contribution to the nucleon susceptibility has not been discussed previously. It has a link, through the relation (17), to the optical potential for the $\sigma$ propagation, which reduces the sigma mass in the medium.

## 2.2 Effect of the two pion propagator

In order to illustrate the coherence of this approach we will now extend the previous description to incorporate the effect of the two-pion propagator, $G$, which affects the nucleon susceptibility in the following way. The sigma propagator is renormalized by the $\sigma$ coupling to two-pion states, as discussed in [4]. At zero four-momentum we have :

$$- D_\sigma = \frac{1}{m_\sigma^2 \,+\, 3\,\lambda\,(m_\sigma^2 - m_\pi^2)\,\frac{G}{1-3\lambda G}} = \frac{1 - 3\,\lambda\,G}{m_\sigma^2 \,-\, 3\lambda\, m_\pi^2\, G} \simeq \frac{1}{m_\sigma^2} \,-\, \frac{3\,G}{2\,f_\pi^2} \tag{20}$$

where $\lambda = (m_\sigma^2 \,-\, m_\pi^2)/2\,f_\pi^2$ and both $D_\sigma$ and $G$ are taken at zero four-momentum. In the last term we have restricted to the one pion loop level. We stress that this expression only holds for the sigma, chiral partner of the pion, which is not a chiral invariant field. It does not apply to the scalar field responsible for the nuclear binding which has to be a scalar invariant (that we have denoted $s$) and which is weakly coupled to two-pion states, while the $\sigma$ is strongly coupled. Therefore this treatment is done for illustration purpose and not for an application to nuclear physics.

The medium correction to $D_\sigma$ from the coupling of the $\sigma$ to $2\pi$ states is :

$$\Delta D_\sigma \;=\; \frac{3\,\Delta G}{2\,f_\pi^2}, \tag{21}$$

where $\Delta G$ is the in-medium modification of the two-pion propagator. In $\Delta G$, to lowest order, one and only one of the two pions of the two-pion propagator has to be dressed by one p-h bubble. It is again possible to interpret the corresponding modification of the sigma propagator as representing a $\sigma N$ scattering amplitude, $T^\pi_{\sigma N}$, in which the sigma interacts with the nucleon pion cloud :

$$T^\pi_{\sigma N} = \frac{3\,m_\sigma^4}{2\,f_\pi^2}\,\frac{\Delta G}{\rho_S}. \tag{22}$$

This is to be compared to the nucleon scalar susceptibility from the pion cloud, which is [4] :

$$\chi^N_S = \frac{3\,\Delta G}{\rho_S}\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^4}. \tag{23}$$

The proportionality factor between the susceptibility (23) and the $\sigma N$ scattering amplitude (22) is the same as previously, $2\,Q_S^{s2}/g_S^2$. We find again that this relation holds not only at the level of p-h excitations but also for a single nucleon, at the level of the nucleonic excitations which in this specific case are of the pionic type.
In summary we have seen that the mixing between the quark density fluctuations and the nucleon ones implies that the response of a probe which couples to the nucleonic density fluctuation is proportional to the QCD scalar response. This includes also the nucleonic contribution to these responses. As an example we have shown that the chiral dropping of the sigma mass in the medium has a counterpart in the form of a contribution of the scalar meson to the QCD scalar susceptibility of the nucleon.

# 3 Connection with lattice data

It is now interesting to connect our results to the available lattice simulations of the evolution of the nucleon mass with the pion mass, equivalently the quark mass. At present they do not cover the physical region but only the region beyond $m_\pi \simeq 400\,MeV$. The derivative $\partial M_N/\partial m_\pi^2 = \sigma_N/m_\pi^2$ provides the nucleon sigma commutator. In turn the derivative of $\sigma_N$ leads to the susceptibility. Both quantities are strongly influenced by the pion cloud contribution which has a non-analytic behavior in the quark mass, preventing a polynomial expansion in this quantity. However the pionic self-energy contribution to the nucleon mass, $\Sigma_\pi$, has been separated out in ref. [9] in a model dependent way with different cut-off forms for the pion loops (gaussian, dipole, monopole) with an adjustable parameter $\Lambda$. The remaining part is expanded in terms of $m_\pi^2$ as follows:

$$M_N(m_\pi^2) = a_0 \,+\, a_2\, m_\pi^2 \,+ a_4\, m_\pi^4 \,+\, \Sigma_\pi(m_\pi). \tag{24}$$

The best fit value of the parameter $a_4$ which fixes the susceptibility shows little sensitivity to the shape of the form factor, with a value $a_4 \simeq -0.5\,GeV^{-3}$ while $a_2 \simeq 1.5\,GeV^{-1}$ (in a previous work [10] smaller values of $a_2$ and $a_4$ were given : $a_2 \simeq 1\,GeV^{-1}$ and $a_4 \simeq -0.23\,GeV^{-3}$). We can infer the non-pionic pieces of the sigma commutator and of the susceptibility from the expansion (24) :

$$\sigma_N^{non-pion} = m_\pi^2\,\frac{\partial M}{\partial m_\pi^2} \;=\; a_2\, m_\pi^2 \,+\, 2\, a_4\, m_\pi^4 \;\simeq 29\,MeV\,. \tag{25}$$

It is largely dominated by the $a_2$ term. The corresponding value for $a_2 \simeq 1\,GeV^{-1}$ is $\sigma_N^{non-pion} = 20\,MeV$.

In turn the nucleon susceptibility is :

$$\chi_S^{N,non-pion} \;= 2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\,\frac{\partial}{\partial m_\pi^2}\left(\frac{\sigma_N^{non-pion}}{m_\pi^2}\right) \;=\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\,4\,a_4 \;\simeq -5.4\,GeV^{-1} \tag{26}$$

The non-pionic susceptibility is found with a negative sign, as expected from the scalar meson term. In ref. [9] however, the negative sign is interpreted differently. It is attributed to possible deviations from the Gellman-Oakes-Renner (GOR) relation which links quark and pion masses.

It is then interesting to test if the empirical values from the lattice are compatible with a pure scalar meson contribution. We thus tentatively make the following identifications :

$$Q_S^s \; = \frac{\langle \bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2} \; = \frac{\sigma_N^{non-pion}}{(2\,m_q)} \simeq 2.4, \tag{27}$$

with $2\,m_q = 12\,MeV$ (taking $a_2 \simeq 1\,GeV^{-1}$ one would get $Q_S^s = 1.66$). It is interesting to translate this number into the value of the mean scalar field in the nuclear medium which, to leading order in density, is :

$$-\,\bar{s} = \frac{g_s\,\rho_S}{m_\sigma^2} = \frac{Q_S^s\,f_\pi\,\rho_S}{\langle \bar{q}q\rangle_{vac}} = \frac{\sigma_N^{non-pion}}{(2\,m_q)}\,\frac{f_\pi\,\rho_S}{\langle \bar{q}q\rangle_{vac}} = \frac{a_2\,+\,a_4\,m_\pi^2}{f_\pi}\,\rho_S\;. \tag{28}$$

At normal density the value $|\bar{s}(\rho_0)| \simeq 21\,MeV$, quite compatible with nuclear phenomenology. The second identification concerns the susceptibility. If the non pionic susceptibility would arise entirely from the scalar field, we should have :

$$\chi_S^{N,non-pion} \; = \; -\,\frac{2\,(Q_S^s)^2}{g_S^2}\,\frac{3\,g_S}{f_\pi} = \frac{\langle \bar{q}q\rangle_{vac}^2}{f_\pi^4}\,4\,a_4 \tag{29}$$

which would give, using the GOR relation :

$$-\,a_4 = \frac{3}{2}\,\frac{(\sigma_N^{non-pion})^2}{g_S\,f_\pi\,{m_\pi}^4} = 3.1\,GeV^{-3}, \tag{30}$$

much larger than the lattice value, $-a_4 = 0.5\,\mathrm{GeV}^{-3}$. Again, taking $a_2 \simeq 1\,GeV^{-1}$ one would get $-a_4 \simeq 1.2\,GeV^{-3}$, still larger in magnitude than the corresponding lattice value ($-0.23\,GeV^{-3}$). Thus the linear $\sigma$ model which fails to account for the saturation properties, due to the excess attraction produced by the chiral softening of the sigma mass, also leads to too large a susceptibility from the scalar meson. In fact in this work we show that the two problems are linked. Some mechanism suppresses the chiral softening of the $\sigma$ mass as well as the large nucleonic susceptibility from the scalar meson, incompatible with lattice data. It is indeed likely that the scalar meson is not the only non-pionic contribution. In a previous work [3] we have invoked confinement and the quark meson coupling model (QMC) [11, 12] as a source of cancellation for the chiral softening of the sigma mass. It turns out that it has also a cancelling effect in the nucleon scalar susceptibility. Indeed, for 3 valence quarks confined in a bag of radius R, Guichon [13] derived $\chi_S^{N,bag} \simeq +0.25\,R \simeq 1\,GeV^{-1}$, for a value $R = 0.8\,fm$. Translated into the parameter $a_4$, one has $a_4^{bag} \simeq +0.1\,GeV^{-3}$. Contrary to the other components which are negative (of paramagnetic nature), it has a positive sign (of the diamagnetic type, linked to quark-antiquark excitations). The bag susceptibility indeed produces a mild cancellation effect.

It is then natural to try to extend the linear sigma model description so as to incorporate other effects than the scalar meson ones (or chiral symmetry breaking effects), for instance those arising from confined valence quarks.

# 4 Generalization and implications for nuclear physics

## 4.1 General relation

In view of the limitations of the linear sigma model discussed previously, a more general approach is desirable. The aim is to link the response of the nuclear medium to the scalar nuclear field and the QCD responses, in such a way that both quantitites include all the components of the individual nucleon contribution whatever their origin. Of course it is not possible to achieve this goal without some assumptions on the nature of the probe. We keep the basic assumption that the scalar field which couples to the nucleons, couples to the quarks of the nucleon condensate, as is the case in the linear sigma model. Thus its presence can induce a readjustement of the quark structure of the nucleon, that we evaluate in the way described below.

Consider a nuclear medium with a scalar nucleon density $\rho_S$. By definition the response of this medium to a scalar field which couples to the nucleon scalar density fluctuations (with a unit coupling constant) is the change in the nucleon scalar density for a small change of the nucleon mass. It is $\Pi_S = (\partial\rho_S/\partial M_N)_\mu$, the derivative being taken at constant chemical potential. With a coupling constant $g_S$ this result should be multiplied by $g_S^2$. In the free Fermi gas case this derivative leads the the quantity $-2\,Mp_F/\pi^2$, the free Fermi gas response. For nucleons interacting via $\sigma$ and $\omega$ exchange, the expression of the scalar nucleon density is :

$$\rho_S = \int \frac{4\,d^3p}{(2\pi)^3}\,\frac{M_N^*}{E_p^*}\,\Theta\left(\mu\,-\,E_p^*\,-\,\frac{g_\omega^2}{m_\omega^2}\,\rho\right). \tag{31}$$

where $M_N^* \;=\; M_N\,(1+\bar{s}/f_\pi)$ is the nucleon effective mass, linked to the mean scalar field $\bar{s}$ and $E_p^* \;=\; \sqrt{p^2\;+\;M_N^{*2}}$. The mean field $\bar{s}$ is obtained from the minimization equation of the energy density $\epsilon$ :

$$\frac{\partial\varepsilon}{\partial\bar{s}} = g_S\,\rho_S\;+\;V'(\bar{s}) = 0\,. \tag{32}$$

It is then possible to check that the derivative of the scalar nucleon density with respect to the nucleon mass leads to the full RPA scalar polarization propagator, $\Pi_S$, as defined in eq. (8). In this expression of the response as the derivative of the nucleon density the nucleon structure is not incorporated. It only includes the effect of the nuclear excitations and not that of the nucleonic ones. In order to include them we have to account for the internal nucleon structure, *i.e.*, the quark structure. It is the quark medium, and not only the nucleon one, which responds to the same excitation, *i.e.*, to the modification of the nucleon mass $\delta M_N$. Accordingly we make the following conjecture, writing the full response $\mathcal{R}_S^{\mathcal{A}}$ as :

$$\mathcal{R}_S^{\mathcal{A}} = \;\frac{1}{2\,Q_S}\,\left(\frac{\partial\rho_S^q}{\partial M_N}\right)_\mu \tag{33}$$

where $\rho_S^q$ is the quark scalar density and the factor $1/2\,Q_S$ in front of the derivative is put for normalization purpose. Each nucleon containing a scalar number of quarks $2\,Q_S = \sigma_N/m_q$, the scalar density of quarks is $\rho_S^q = 2\,Q_S\,\rho_S$. The derivative involves two

terms :

$$\mathcal{R}_{\mathcal{S}}^{\mathcal{A}} = \frac{1}{Q_S}\left(\frac{\partial}{\partial M_N}(Q_S\,\rho_S)\right)_\mu = \left(\frac{\partial\rho_S}{\partial M_N}\right)_\mu + \frac{\rho_S}{2\,Q_S^2}\frac{\partial Q_S}{\partial m_q}. \tag{34}$$

In the last term we have replaced the derivative with respect to the nucleon mass by the one with respect to the quark mass, with $\partial M_N/\partial m_q = 2\,Q_S$, which introduces the nucleon susceptibility $\chi_S^N$. The overall result writes :

$$\mathcal{R}_{\mathcal{S}}^{\mathcal{A}} = \frac{\chi_S^N}{2\,Q_S^2}\,\rho_S + \Pi_S. \tag{35}$$

The interpretation of this equation is clear. This decomposition is obvious and analogous to the one of eq. (5). The term linear indensity represents the individual nucleon response from the nucleonic excitations, while the term in $\Pi_S$ embodies nuclear excitations. The new information is that the single nucleon response is proportional to the QCD one, $\chi_S^N$, with the same proportionality factor $1/(2Q_S^2)$, as was found previously for the nuclear excitations. All in all, the eq. (35) writes :

$$\mathcal{R}_{\mathcal{S}}^{\mathcal{A}} = \frac{1}{2\,Q_S^2}\,\chi_S^A \tag{36}$$

where $\chi_S^A$ represents the total scalar QCD susceptibility of the nuclear medium (vacuum value substracted) and both members incorporate the individual nucleon contribution. This results holds for a unit coupling constant. For a coupling constant $g_S$ (as is the case for the nuclear scalar field) the r.h.s should be multiplied by $g_S^2$. Accordingly the corresponding $\sigma N$ amplitude is :

$$T_{\sigma N} = \frac{\chi_S^N g_S^2}{2\,Q_S^2}. \tag{37}$$

We will now comment this result and we then will apply it to the problem of the propagation of the scalar field which mediates the nuclear attraction. Our relation (36) has a close resemblance to the previous one, (18), derived in the linear sigma model but here we do not inquire about the origin of the terms, $\chi_S^N$ and $Q_S$. With the values of the linear sigma model for these quantities we recover the previous result of this model.

Our relation (37) is also very similar to the one of the quark-meson coupling model [11]. In QMC, the bag positive susceptibility manifests itself in the form of a repulsive interaction in the propagation of the scalar nuclear field. The corresponnding scattering amplitude is related to the bag suceptibility by a relation identical to our eq. (37), but only bag quantities appear and the scalar charge entering this relation is that of the bag, which is $Q_S^{bag} \simeq 0.7$. As QMC does not incorporate the chiral potential which implies the three-scalar coupling, only the repulsive three-body interaction from the bag structure enters. Our expression (37) thus covers the two extreme situations, when the nucleon mass originates totally from the condensate as is the case in the $\sigma$ model, or when it is only due to confinement. It is legitimate to believe that it is able to describe a more general situation with a mixed origin.

## 4.2 Illustration in a hybrid model of the nucleon

In the following we will illustrate the relation (37) in a model of the nucleon proposed by Shen and Toki [14], where the nucleon mass originates in part from its coupling to the condensate and in part from confinement. It consists in the following: three constituant quarks, described in the Nambu-Jona-Lasinio model (NJL), are kept together by a central harmonic force so as to mimick confinement. We have chosen for simplicity the form: $((K/4)(1+\gamma_0)\, r^2$ which leads to analytical results. Although oversimplified the model gives an intuitive picture of the role played by confinement. Denoting M the mass of a free constituant quark and E that of the bound one, the nucleon mass is given :

$$M_N = 3\,E = 3\left(M + \frac{3}{2}\sqrt{\frac{K}{E+M}}\right) . \tag{38}$$

It is increased as compared to the value, $3M$, for three independent constituant quarks.. The nucleon scalar charge, $Q_S$, is :

$$Q_S = \frac{3}{2}\,\frac{\partial E}{\partial m_q} = \frac{3}{2}\,\frac{\partial E}{\partial M}\frac{\partial M}{\partial m_q} \tag{39}$$

with :

$$\frac{\partial E}{\partial M} = c_S = \frac{E+3M}{3E+M} . \tag{40}$$

As $E > M$, $c_S < 1$, the nucleon scalar charge is reduced as compared to a collection of three independent constituant quarks. The nucleon scalar susceptibility, $\chi_S^N$, given by the next derivative, is composed of two terms arising respectively from the derivative of $c_S$ and from that of $\partial M/\partial m$. The second part leads to the susceptibility, $\chi_S^q$, of a free constituant quark,

$$\begin{aligned} \chi_S^N = \frac{\partial Q_S}{\partial m_q} &= \frac{3}{2}\left[\frac{\partial c_S}{\partial M}\left(\frac{\partial M}{\partial m_q}\right)^2 + c_S\,\frac{\partial^2 M}{\partial^2 m_q^2}\right] \\ &= \frac{3}{2}\,\frac{\partial c_S}{\partial M}\left(\frac{\partial M}{\partial m_q}\right)^2 + 3\,c_S\,\chi_S^q \end{aligned} \tag{41}$$

with:

$$\frac{\partial c_S}{\partial M} = \frac{24\,(E^2 - M^2)}{(3E+M)^3} . \tag{42}$$

Notice that this last derivative is positive since $E > M$ and that it vanishes in the absence of confining force, when $E = M$. Therefore the first part of the expression of $\chi_S^N$ represents the part of the susceptibility originating in confinement and, as in QMC, it is positive.

The scalar coupling constant $g_S$ is linked to the derivative of the nucleon mass with respect to the mean scalar field $\bar{s}$:

$$g_S = 3\,\frac{\partial E}{\partial \bar{s}} = 3\,\frac{\partial E}{\partial M}\frac{\partial M}{\partial \bar{s}} = 3\,c_S\,g_q \tag{43}$$

where $g_q$ is the corresponding coupling constant for a constituant quark.

The nucleon response to the scalar field originating in confinement, $\kappa_{NS}$, is linked to the second derivative of the nucleon mass with respect to the scalar field :

$$\kappa_{NS} = 3\,\frac{\partial^2 E}{\partial \bar{s}^2} = 3\,\frac{\partial c_S}{\partial M}\left(\frac{\partial M}{\partial \bar{s}}\right)^2 . \tag{44}$$

The ratio between the part of the nucleon scalar susceptibility which is due to confinement and $\kappa_{NS}$ is

$$r = \frac{1}{2}\,\frac{(\frac{\partial M}{\partial m_q})^2}{(\frac{\partial M}{\partial s})^2} = 2\,\frac{Q_S^2}{g_S^2}, \tag{45}$$

the same ratio as was previously found.

As for the scalar $\sigma N$ amplitude from the tadpole term, $T_{\sigma N} = 3\,g_S/f_\pi$, it should be compared to the other part of the susceptibility. We define $r'$ as the corresponding ratio through :

$$\frac{3}{2}\,c_S\,\frac{\partial^2 M}{\partial m_q^2} = r'\,\frac{3\,g_S}{f_\pi}\,. \tag{46}$$

In the semi-bosonized version of the NJL model we have :

$$\frac{\partial M}{\partial m_q} = -2\,\frac{g_q\,\langle \bar{q}q\rangle_{vac}}{f_\pi\,m_\sigma^2} \tag{47}$$

and

$$\frac{\partial^2 M}{\partial m_q^2} = 2\,\chi_S^q = \frac{2\,g_q\,\langle \bar{q}q\rangle_{vac}^2}{f_\pi^3\,m_\sigma^4} \tag{48}$$

in such a way that the ratio $r'$ becomes :

$$r' = \frac{2\,\langle \bar{q}q\rangle_{vac}^2}{f_\pi^2\,m_\sigma^4} = \frac{2\,Q_S^2}{g_S^2} \equiv r. \tag{49}$$

Since the same ratio applies to the two parts, $r' \equiv r$, it can be factorized so as to obtaIn the relation (37), which is thus confirmed in this hybrid model.
Numerically a value of the ratio $E/M \simeq 2.1$, which gives $c_S \simeq 0.7$, leads to a reasonable value for $g_A$. It results in a value of the dimensionless parameter $C = (f_\pi^2/2M)\,\kappa_{NS} \simeq 0.1$, while the value needed to account for the saturation properties in the framework of chiral models is $C \simeq 1$ [15]. Even if it fails to account for the numerical value this model has the merit to confirm the validity of the relation between the QCD response and the one to the nuclear scalar field in a situation where confinement enters and to illustrate the role played by confinement, with the introduction of a positive component in the susceptibility which opposes an increase of the nuclear scalar field.

We can now turn to the quantitative applications of the relation (37). Since both the total (non-pionic) nucleonic susceptibility and scalar charge enter in the expression of the (chiral invariant) scalar-nucleon scattering amplitude, it is legitimate to use for these two quantities the phenomenological values obtained from the lattice data. We can therefore infer the medium effects in the propagation of the s field from the lattice results of eq. (25) and (26) as :

$$-\,D_s^{-1} = m_\sigma^2\,+\,\frac{g_S^2}{2\,Q_S^2}\,\chi_S^N\,\rho_S = m_\sigma^2\,+\,g_S^2\,\frac{2\,a_4}{(a_2\,+\,2\,a_4\,m_\pi^2)^2}\,\rho_S \tag{50}$$

where in the first equation only non-pionic quantities enter, hence the introduction of the parameters $a_2$ and $a_4$ which have been defined in the lattice expansion. Numerically, at normal nuclear density, and for a value of the coupling constant $g_S = 10$, the second term on the rhs of the second equation takes the value $0.06\, GeV^2$ (a similar value is found for the other set of parameters $a_2$ and $a_4$). For a sigma mass of $m_\sigma = 0.75\, GeV$, this represents at $\rho_0$ only a 6% decrease of the mass, much less that the chiral dropping alone and in much better agreement with the nuclear phenomenology [3, 15].

# 5 Conclusion

In summary we have studied in this work the interplay between the nuclear responses to probes which couple either to nucleon or to quark scalar density fluctuations. We have found that the two responses are closely related, being proportional to each other. The scaling coefficient involves the scalar charge of the nucleon $Q_S$. Our result holds not only at the level of the nuclear excitations but also at the nucleonic ones such that both responses incorporate the individual nucleon contributions to the nuclear response. Thus the scalar response of a nucleon to the nuclear scalar field is proportional to its QCD scalar susceptibility. We have confirmed this relation in the Shen and Toki model of the nucleon where its mass arises in part from the coupling to the condensate and in part from confinement.

One application of this relation concerns a free nucleon. The $\sigma N$ amplitude from the tadpole term has a counterpart in the QCD scalar susceptibility in the form of a negative contribution beyond the pionic one. We have tested its existence in the lattice results on the nucleon mass evolution with the pion mass, as analyzed in ref. [9]. The expansion of ref. [9] is indeed compatible with a negative component for the non-pionic susceptibility. However the magnitude does not fit, indicating the existence of other components, with a cancelling effect. In fact a similar cancellation has to occur in the saturation problem of nuclear matter. The $3\sigma$ coupling is responsible for a lowering of the $\sigma$ mass, which produces too much attraction at large densities and destroys saturation. It has to be compensated. In the optics of the present work, the two effects are related. The full $\sigma N$ scattering amplitude being proportional to the susceptibility, a cancelling effect in the amplitude is automatically reflected in the susceptibility. Confinement may be invoked as a natural mechanism for cancellation.

The existence of a link between QCD and nuclear physics quantities allows the derivation of parameters of the $\sigma\omega$ model from the lattice results on the nucleon mass dependence on the quark mass. This procedure leads to a mean scalar field $|\bar{s}(\rho_0)| \simeq 20\, MeV$. In our approach the nucleon response to this field can also be derived from the lattice data. Altogether this method leads to a satisfactory description of the nuclear matter saturation properties.

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